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ON THE REGULARITIES OF COMPOSITE HEAT TRANSFER

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Abstract—The regularities of heat transfer by conduction, convection and radiation are considered on the basis of the common law of energy transfer. An analysis of the relations obtained is given for a number of particular cases of heat transfer. The interdependence between heat transfer processes by conduction and radiation for the spherical layer of the medium is established.

Résumé—Les caractères de régularité de la transmission de chaleur par conduction, convection et rayonnement sont considérés à partir de la loi courante de la transmission de chaleur. Une étude des relations obtenues est faite pour un certain nombre de cas particuliers de transmission de chaleur. L'interdépendance des phénomènes de transmission de chaleur par conduction et rayonnement est établie dans le cas d'un milieu en couche sphérique.

Zusammenfassung—Es werden die Besonderheiten der Wärmeübertragung durch Leitung, Konvektion und Strahlung auf der Grundlage des gemeinsamen Gesetzes der Energieübertragung betrachtet. Für mehrere Spezialfälle der Wärmeübertragung wird eine Analyse der erhaltenen Beziehungen gegeben. Diese gegenseitige Abhängigkeit zwischen Wärmeübertragungsprozessen durch Leitung und Strahlung wird für die sphärische Schicht eines Mediums behandelt.

Аннотация—На основе единого закона переноса энергии рассматриваются закономерности переноса тепла кондукцией, конвекцией и излучением. Дан анализ полученных соотношений для ряда частных случаев теплообмена. Установлена связь процессов передачи тепла кондукцией и радиацией для сферического слоя среды.

By composite heat transfer one understands simultaneous heat transfer by conduction, convection and radiation in a solid medium.

The vector of heat transfer by conduction $\mathbf{q}_{cond}$ is determined on the basis of the energy transfer law [1] by the following expression:

$$\mathbf{q}_{cond} = -a \text{ grad } \epsilon \tag{1}$$

where a is the coefficient of molecular energy transfer and ϵ is the density of the medium's molecular energy.

As it is already known the density of the medium's molecular energy has the value:

$$\epsilon = \rho\bar{c}T$$

where ρ is the mass density of the medium, $\bar{c}$ is the average heat capacity of the medium and T is the molecular ambient temperature.

This expression at constant ρ and $\bar{c}$ enables equation (1) to be written as follows:

$$\mathbf{q}_{cond} = -a\rho\bar{c} \text{ grad } T.$$

Let us designate the product $a\rho\bar{c}$ by λ and call it the coefficient of thermal conductivity. Then

$$\mathbf{q}_{cond} = -\lambda \text{ grad } T. \tag{2}$$

It is evident that

$$a = \frac{\lambda}{\rho\bar{c}}$$

i.e. the coefficient of molecular energy transfer is equal to the thermal diffusivity coefficient of the medium.

For the vector $\mathbf{q}_{conv}$ of heat transfer by convection the following equation will hold true:

$$\mathbf{q}_{conv} = \rho\bar{c}T\mathbf{w} \tag{3}$$

where $\mathbf{w}$ is the vector of the medium's motion rate.

When heat is transferred by radiation in a grey medium photons having the velocity of light distribution c are absorbed and emitted by the molecules of the solid medium that causes their irregular motion.

The law of energy transfer is applied to such a motion and an expression analogous to equation (1) can be written for the vector $\mathbf{q}_r$ of heat transfer by radiation:

$$\mathbf{q}_r = -a_r \operatorname{grad} \epsilon_r \quad (4)$$

where a_r is the coefficient of the radiative energy transfer, ϵ_r is the density of the radiant energy.

Close by the heat receiving surfaces the irregular motion of the photons causes the distribution of the intensities, as shown in Fig. 1. The value

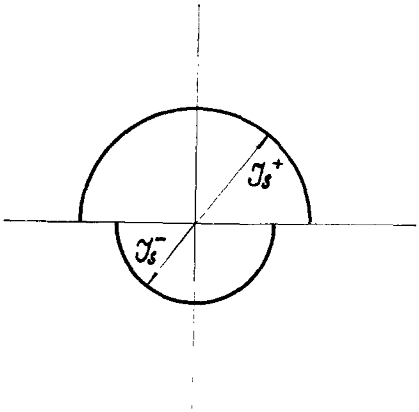

FIG. 1. Distribution of the radiation intensity near a heat receiving surface.

a_r for such a distribution will be approximately equal to:

$$a_r = \frac{1}{4} c l_s$$

where l_s is the average length of free paths of the photons.

The value l_s is bound with the absorption coefficient by the known relation:

$$l_s = \frac{1}{k}. \quad (5)$$

Therefore

$$a_r = \frac{1}{4}\frac{c}{k}. \quad (6)$$

The density of the radiant energy in the case considered is:

$$\epsilon_r = \frac{\int_{2\pi} J_s^+ \cdot d\omega}{c} + \frac{\int_{2\pi} J_s^- \cdot d\omega}{c} = \frac{4\sigma_0 \bar{T}_r^4}{c} \quad (7)$$

where J_s^+ and J_s^- are the radiation intensities in the corresponding semispaces, $d\omega$ is the elementary spatial angle and $\bar{T}_r$ is the average radiant temperature.

With the aid of equations (6) and (7) we can rewrite equation (4) as

$$\mathbf{q}_r = -\frac{\sigma_0}{k} \operatorname{grad} \bar{T}_r^4. \quad (8)$$

The molecular temperature T is bound with the radiant temperature T_r by the equation derived in [1]:

$$4\sigma_0 k(T_r^4 - T^4) = -\operatorname{div} \mathbf{q}_r. \quad (9)$$

Taking the expression for $\mathbf{q}_r$ into account and considering the value k as constant enable the above equation to be rewritten as

$$T_r^4 - T^4 = \frac{1}{4k^2} \nabla^2 T_r^4. \quad (10)$$

A consideration of equations (2, 3, 8 and 10) gives the conclusion that the processes of heat transfer by conduction, convection and radiation are interconnected and the change of the molecular temperature field of the medium will cause a certain change of its radiant temperature field.

It is known that the hydrodynamic conditions of the process of the medium's motion essentially influence conduction and convection. Radiation too will be essentially influenced by these conditions.

It should be noted that till now the interconnection of the conduction and convection processes on the one hand and of radiation on the other has been insufficiently taken into account and the calculations of composite heat transfer have usually been made without regard for this interconnection,

We shall now consider heat transfer between the grey medium and the heat receiving surface H_w.

Let the surface H_w with the constant temperature T_w be a plane with the normal $\bar{s}$ (Fig. 2).

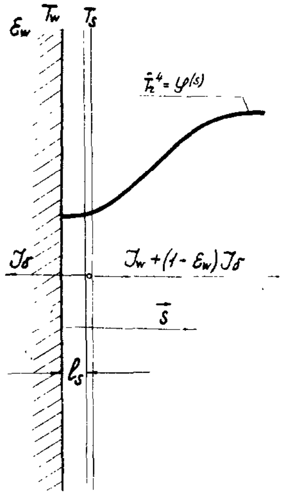

FIG. 2. A calculation scheme of heat transfer by radiation from a wall into the surrounding medium.

The radiant energy in the medium close to the wall, having a layer of thickness l_s moves in the direction **s** in such a way that its interaction with the medium seems to be absent, in consequence of which the radiant temperature in this layer has a constant value.

In the medium's kernel, which is adjacent to the layer close to the wall, the radiant energy interacts with the medium thus having a variable value.

Near the surface H_w, which is isothermal, equation (10) will be of the type:

$$T_r^4 - T^4 = \frac{1}{4k^2}\frac{d^2\bar{T}_r^4}{ds^2}. \qquad (11)$$

Near the plane H_w the value $\bar{T}_r^4$ is determined by the co-ordinate s alone.

The curvature radius of the curve $\bar{T}_r^4 = \varphi(s)$ is equal to infinity on the boundary between the layer close to the wall and the medium's kernel, since in this layer the curve turns into a straight line parallel to the direction **s**.

The expression for the curvature radius of the curve considered is:

$$R = \frac{[1 + (d\bar{T}_r^4/ds)^2]^{3/2}}{(^2d\bar{T}_r^4/ds^2)}.$$

If $R = \infty$, then

$$\frac{d^2\bar{T}_r^4}{ds^2} = 0.$$

Thus equation (11) gives:

$$\bar{T}_r = T.$$

This means that on the boundary between the layer close to the wall and the medium's kernel there exists a layer where the radiant and molecular temperatures are equal, i.e. a layer which is in a radiant equilibrium state. It will be referred to as an equilibrium layer. The existence of the equilibrium layer has been proved experimentally [2, 3].

Suppose the radiations in the semispaces on the left and on the right of the equilibrium layer to be isotropic.

Let the radiation intensity on the left of the equilibrium layer have the value J_δ. Then the radiation intensity on the right of this layer will be equal to $J_w + (1 - \epsilon_w)J_\delta$, where J_w is the intensity of the wall radiation which is considered to be isotropic as well.

The vector magnitude of heat transfer by radiation is given by:

$$q_r = \pi\{J_\delta - [J_w + (1 - \epsilon_w)J_\delta]\} = \pi(\epsilon_w J_\delta - J_w).$$

The equation for the radiant equilibrium layer on the assumption that the radiation is isotropic will have the form:

$$J_\delta + [J_w + (1 - \epsilon_w)J_\delta] = 2\frac{\eta}{k} = 2J_0$$

or

$$(2 - \epsilon_w)J_\delta + J_w = 2\frac{\sigma_0 T_\delta^4}{\pi}$$

where T_δ is the molecular temperature of the equilibrium layer.

Solving the previous equation for J_δ gives:

$$J_\delta = \frac{2}{\pi(2 - \epsilon_w)}\sigma_0 T_\delta^4 - \frac{\epsilon_w}{\pi(2 - \epsilon_w)}\sigma_0 T_w^4.$$

Introducing this value into the expression for q_r results in:

$$q_r = \left(\frac{2\epsilon_w}{2 - \epsilon_w}\right)\sigma_0 T_\delta^4 - \left(\frac{\epsilon_w^2}{2 - \epsilon_w}\right)\sigma_0 T_w^4 - \epsilon_w \sigma_0 T_w^4$$

We conclude from this expression that:

$$q_r = \frac{\sigma_0}{(1/\epsilon_w) - \frac{1}{2}} (T_\delta^4 - T_w^4) \qquad (12)$$

which determines heat transfer between the medium and the surface H_w.

We consider now the simplest problems of heat transfer in the immovable grey solid medium assuming that heat transfer by conduction is negligibly small as compared with heat transfer by radiation.

Two solid planes are taken at constant temperatures T_1 and T_2, where $T_1 > T_2$ (Fig. 3). Let the emissivities of the planes be ϵ_1 and ϵ_2. Suppose a layer of grey solid medium of thickness l with a constant absorption coefficient k and the coefficient of self-radiation η to be between the planes.

We shall determine the vector $\mathbf{q}_r$ of heat transfer by radiation assuming that the medium is in a radiant equilibrium state, i.e. the amount of radiant energy, absorbed by an elementary volume of the medium for a certain period of time is equal to the amount of energy emitted by the same medium volume for the same period of time.

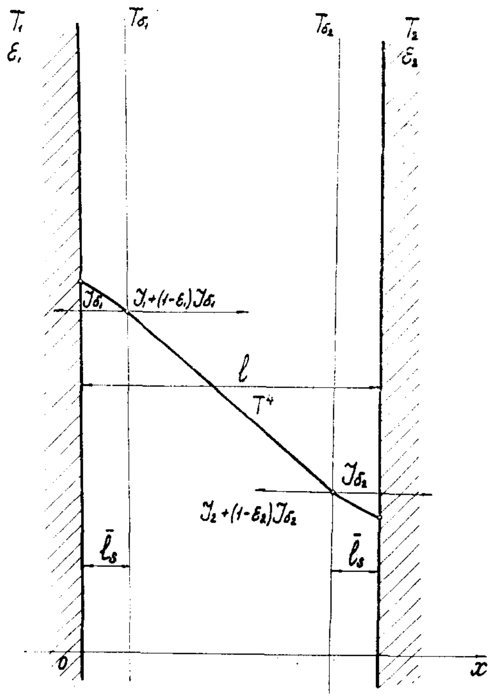

FIG. 3. A calculation scheme of heat transfer between two walls with temperatures T_1 and T_2.

The radiant equilibrium state of the medium is determined by equation (1):

$$\operatorname{div} \mathbf{q}_r = 4\sigma_0 k(T_r^4 - T^4). \qquad (13)$$

It follows from this equation that

$$\bar{T}_r = T$$

i.e. the molecular and the average radiant temperatures have the same value at the radiant equilibrium of the medium.

Substituting equation (8) into the equation of the radiant equilibrium of the medium

$$\operatorname{div} \mathbf{q}_r = 0$$

gives

$$\nabla^2 T^4 = 0. \qquad (14)$$

While finding a further solution for the problem we suppose that T is a function of the coordinate x only, i.e. we consider this problem to be one-dimensional. With this assumption

$$\frac{d^2 T^4}{dx^2} = 0. \qquad (15)$$

It should be noted that the equation will not hold true for medium layers of thickness $\bar{l}_s$ close to the wall since the photons in these layers do not interact on the whole with the molecular medium and give corresponding values for the radiation intensities.

If we assume the medium radiation in the semi-spaces which are adjacent to the equilibrium layers to be isotropic then the photon fluxes in these layers give the radiation intensities shown in Fig. 3.

The boundary conditions for the problem in question may be written as follows:

$$|T|_{x=\bar{l}_s} = T_{\delta_1}; \quad |T|_{x=l-\bar{l}_s} = T_{\delta_2}.$$

The solution for equation (15) and its agreement with the formulated boundary conditions gives:

$$T^4 = -\frac{T_{\delta_1}^4 - T^4{}_{\delta_2}}{l - 2\bar{l}_s}(x - \bar{l}_s) + T_{\delta_1}^4. \qquad (16)$$

For a one-dimensional problem the following expression will hold true:

$$q_r = -\frac{\sigma_0}{k}\frac{\mathrm{d}\bar{T}_r^4}{\mathrm{d}x}$$

and at the radiant equilibrium of the medium

$$q_r = -\frac{\sigma_0}{k}\frac{\mathrm{d}T^4}{\mathrm{d}x}. \qquad (17)$$

Having found the derivative $\mathrm{d}T^4/\mathrm{d}x$ from equation (16) we substitute it into equation (17) and have:

$$q_r = \sigma_0 \frac{T_{\delta_1}^4 - T_{\delta_2}^4}{k(l - 2\bar{l}_s)}. \qquad (18)$$

In the equilibrium layers the following equations are valid:

$$J_1 - \epsilon_1 J_{\delta_1} = \frac{q_r}{\pi} \qquad (19)$$

$$\epsilon_2 J_{\delta_2} - J_2 = \frac{q_r}{\pi}. \qquad (20)$$

The radiant equilibrium state of the medium in the equilibrium layers enables the following equations to be written:

$$2\pi\left\{2\,[J_1 + (1 - \epsilon_1) J_{\delta_1}] - \frac{q_r}{\pi}\right\} = 4\pi\frac{\sigma_0 T_{\delta_1}^4}{\pi} \qquad (21)$$

$$2\pi\left\{2[J_2 + (1 - \epsilon_2) J_{\delta_2}] + \frac{q_r}{\pi}\right\} = 4\pi\frac{\sigma_0 T_{\delta_2}^4}{\pi}. \qquad (22)$$

From the system of equations (18–22) we get

$$q_r = \frac{\sigma_0(T_1^4 - T_2^4)}{(1/\epsilon_1) + (1/\epsilon_2) - 1 + k(l - 2\bar{l}_s)}. \qquad (23)$$

This formula is valid only when $l > 2\bar{l}_s$. If $l \leqslant 2\bar{l}_s$ then instead of formula (23) one should use the known formula:

$$q_r = \frac{\sigma_0(T_1^4 - T_2^4)}{(1/\epsilon_1) + (1/\epsilon_2) - 1} \qquad (24)$$

which is a particular case of equation (23) for $l = 2\bar{l}_s$.

In the presence of equation (5) equation (23) can be rewritten as:

$$q_r = \frac{\sigma_0(T_1^4 - T_2^4)}{(1/\epsilon_1) + (1/\epsilon_2) - 1 + (kl - 2)}. \qquad (25)$$

If $\bar{l}_s \ll l$ equation (23) then becomes:

$$q_r = \frac{\sigma_0(T_1^4 - T_2^4)}{(1/\epsilon_1) + (1/\epsilon_2) - 1 + kl}. \qquad (26)$$

In the literature equation (26) is given for the problem considered but without indication of the limits to the correctness of this solution.

Two cylindric coaxial surfaces of radii r_1 and r_2 are taken (Fig. 4). Let the emissivities of the internal and external surfaces have the values ϵ_1 and ϵ_2, and their temperatures be constant and equal to T_1 and T_2, respectively, where $T_1 > T_2$.

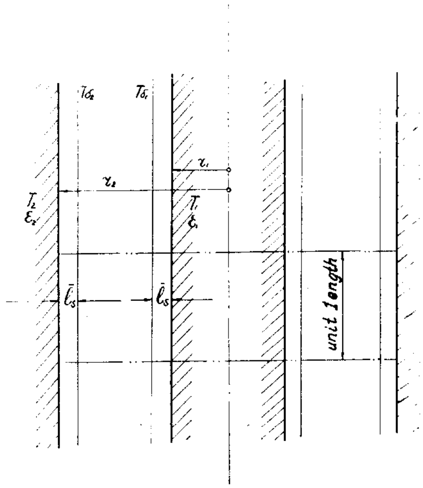

FIG. 4. On the calculation of heat transfer between cylindrical surfaces.

Suppose an absorbing grey medium with the constant absorption coefficient k and the radiation coefficient η to be between the cylindrical walls.

The amount of radiant energy q_r transferred per unit time between unit lengths of the cylindrical surfaces at the radiant equilibrium of the grey medium is to be determined.

The radiant equilibrium state of the medium is given by equation (13).

This equation for a one-dimensional problem has the form:

$$\frac{\mathrm{d}^2T^4}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}T^4}{\mathrm{d}r} = 0. \qquad (27)$$

The equation holds true in the medium cylindrical layer of radii $r_1 + \bar{l}_s$ and $r_2 - \bar{l}_s$.

The boundary conditions for the problem considered are formulated as follows:

$$|T|_{r=r_1+\bar{l}_s} = T_{\delta_1}$$

$$|T|_{r=r_2-\bar{l}_s} = T_{\delta_2}.$$

The solution for equation (27) under these boundary conditions has the form:

$$T^4 = -\frac{T_{\delta_1}^4 - T_{\delta_2}^4}{\ln[(r_2-\bar{l}_r)/(r_1-\bar{l}_r)]} \ln\frac{r-\bar{l}_r}{r_1+\bar{l}_r} + T_{\delta_1}^4 \quad (28)$$

The value q_r is determined from:

$$q_r = (\sigma_0/k)2\pi r(\mathrm{d}T^4/\mathrm{d}r). \quad (29)$$

Having found the product $r(\mathrm{d}T^4/\mathrm{d}r)$ from equation (28), we substitute it into expression (29) and get:

$$q_r = \frac{T_{\delta_1}^4 - T_{\delta_2}^4}{\dfrac{1}{2\pi(\sigma_0/k)} \ln\dfrac{r_2-\bar{l}_r}{r_1+\bar{l}_r}}. \quad (30)$$

Here the molecular temperatures of the equilibrium layers are designated by T_{δ_1} and T_{δ_2}.

Let the unit length of the internal cylindrical surface have the self-radiation q_1.

It is evident that

$$q_1 = \epsilon_1\sigma_0 T_1^4 2\pi r_1.$$

Let the radiation to the interior of the layer through a unit length of the external boundary surface of the internal layer close to the wall have the value q_{δ_1}.

Then for q_r we can write the following expression:

$$q_r = q_1 - \epsilon_1\varphi_{\delta_1}q_{\delta_1} \quad (31)$$

where φ_{δ_1} is the angle coefficient of the external boundary surface of the internal layer close to the wall relative to this very wall.

Let the length unit of the external cylindrical surface have the self-radiation q_2:

$$q_2 = \epsilon_2\sigma_0 T_2^4 \cdot 2\pi r_2.$$

We shall designate the value of radiation into the interior of the layer through a unit length of the internal boundary surface of the external layer close to the wall by q_{δ_2}. Then

$$q_r = \epsilon_2 q_{\delta_2} - q_2. \quad (32)$$

The conditions of radiant equilibrium for the medium in the equilibrium layers enable the equations to be written as follows:

$$2\pi\{2[q_1 + (1-\epsilon_1)\varphi_{\delta_1}q_{\delta_1}] - q_r\} = 4\frac{\sigma_0 T_{\delta_1}^4}{\pi} 2\pi(r_1+\bar{l}_s) \quad (33)$$

$$2\pi\{2[q_2 + (1-\epsilon_2)q_{\delta_2}]\varphi_{2\delta} + q_r\} = 4\pi\frac{\sigma_0 T_{\delta_2}^4}{\pi} 2\pi(r_2-\bar{l}_s). \quad (34)$$

The system of equations (30–34) makes it possible to determine the value q_r.

Solving equations (31) and (32) for q_{δ_1} and q_{δ_2} gives

$$q_{\delta_1} = \frac{q_1 - q_r}{\epsilon_1\varphi_{\delta_1}}$$

$$q_{\delta_2} = \frac{q_r + q_2}{\epsilon_2}$$

Taking into account these equalities from equations (33) and (34) we shall have:

$$T_{\delta_1}^4 = \frac{q_1 + (1-\epsilon_1)[(q_1-q_r)/\epsilon_1] - (q_r/2)}{2\pi\sigma_0(r_1+\bar{l}_s)}$$

$$T_{\delta_2}^4 = \frac{\{q_2 + (1-\epsilon_2)[(q_r+q_2)/\epsilon_2]\}\varphi_{2\delta} + (q_r/2)}{2\pi\sigma_0(r-\bar{l}_s)}$$

Substituting these expressions into equation (30) and performing simple transformations we obtain:

$$q_r = \left[\left(\frac{q_1}{\epsilon_1(r_1+\bar{l}_s)}\right) - \left(\frac{q_2\varphi_{2\delta}}{\epsilon_2(r_2-\bar{l}_s)}\right)\right] \Big/ \left[\left(\frac{1/\epsilon_1 - \frac{1}{2}}{r_1+\bar{l}_r}\right) + k\ln\left(\frac{r_2-\bar{l}_s}{r_1+\bar{l}_s}\right) + \left(\frac{(1/\epsilon_2 - 1)\varphi_{2\delta} + \frac{1}{2}}{r_2-\bar{l}_s}\right)\right]$$

or taking into account the expressions for q_1 and q_2:

$$q_r = \left[\frac{\sigma_0 T_1^4 \cdot 2\pi r_1}{r_1+\bar{l}_s} - \frac{\sigma_0 T_2^4 \cdot 2\pi r_2\varphi_{2\delta}}{r_2-\bar{l}_s}\right] \Big/ \left[\frac{1/\epsilon_1 - \frac{1}{2}}{r_1+\bar{l}_s} + k\ln\frac{r_2-\bar{l}_s}{r_1+\bar{l}_s} + \frac{(1/\epsilon_2 - 1)\varphi_{2\delta} + \frac{1}{2}}{r_2-\bar{l}_s}\right]. \quad (35)$$

For the determination of $\varphi_{2\delta}$ we shall assume that

$$T_1 = T_2.$$

Then $q_r = 0$ and

$$\frac{r_1}{r_1 + \bar{l}_s} - \frac{r_2\varphi_{2\delta}}{r_2 - \bar{l}_s} = 0$$

Hence:

$$\varphi_{2\delta} = \left(\frac{r_1}{r_2}\right)\left(\frac{r_2 - \bar{l}_s}{r_1 + \bar{l}_s}\right)$$

Substituting this expression into equation (35) finally gives:

$$q_r = \frac{\sigma_0(T_1^4 + T_2^4)2\pi r_1}{(1/\epsilon_1 + \frac{1}{2}) + (r_1/r_2)(1/\epsilon_2 - 1) + k(r_1 + \bar{l}_s)\ln[(r_2 - \bar{l}_s)/(r_1 + \bar{l}_s)] + \frac{1}{2}[(r_1 + \bar{l}_s)/(r_2 - \bar{l}_s)]} \quad (36)$$

This formula determines heat transfer between cylindrical walls with a grey medium between them and which is in the radiant equilibrium state.

This formula is valid provided that:

$$r_2 > r_1 + 2\bar{l}_s$$

If we suppose that

$$r_2 = r_1 + 2\bar{l}_s$$

then equation (36) will be of the type:

$$q_r = \frac{2\pi r_1\sigma_0(T_1^4 - T_2^4)}{(1/\epsilon_1) + (r_1/r_2)(1/\epsilon_2 - 1)}. \quad (37)$$

This formula is to be used in cases when $r_2 \leqslant r_1 + 2\bar{l}_s$. It coincides with the Christiansen formula.

With the presence of equation (5), equation (36) can be rewritten as:

$$q_r = \frac{\sigma_0(T_1^4 - T_2^4)2\pi r_1}{(1/\epsilon_1 - \frac{1}{2}) + (r_1/r_2)(1/\epsilon_2 - 1) + (kr_1 + 1)\ln[(kr_2 - 1)/(kr_1 + 1)] + \frac{1}{2}[(kr_1 + 1)/(kr_2 - 1)]} \quad (38)$$

If $\bar{l}_s$ is small as compared with r_1 and r_2 then equation (36) becomes:

$$q_r = \frac{\sigma_0(T_1^4 - T_2^4)2\pi r_1}{(1/\epsilon_1 - \frac{1}{2}) + (r_1/r_2)(1/\epsilon_2 - \frac{1}{2}) + kr_1\ln(r_2/r_1)} \quad (39)$$

Let us now consider heat transfer by radiation between concentric spheres of radii r_1 and r_2 (Fig. 5) under conditions which are analogous to those for heat transfer by radiation between cylindrical surfaces.

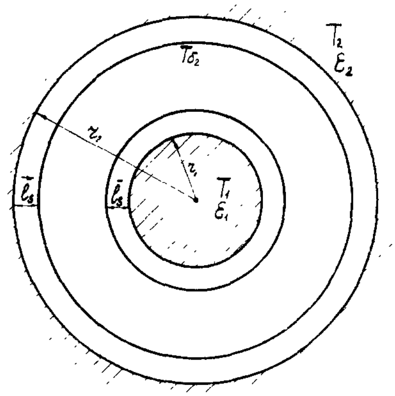

FIG. 5. On the calculation of heat transfer between concentric spherical walls.

The amount of radiant energy transferred per unit time by the sphere surfaces is to be determined.

Solving the one-dimensional equation for the stationary distribution of radiant-energy in the sphere layer gives:

$$Q_r = \frac{4\pi(\sigma_0/k)(T_{\delta_1}^4 - T_{\delta_2}^4)}{1/(r_1 + \bar{l}_s) - 1/(r_2 - \bar{l}_s)}. \quad (40)$$

The self-radiations of the spherical surfaces are:

$$Q_1 = \epsilon_1\sigma_0 T_1^4 4\pi r_1^2$$

$$Q_2 = \epsilon_2\sigma_0 T_2^4 4\pi r_2^2.$$

Considerations analogous to those given above make it possible for the following equations to be derived:

$$Q_r = Q_1 - \epsilon_1\varphi_{\delta_1}Q_{\delta_1} \quad (41)$$

$$Q_r = \epsilon_2 Q_{\delta_2} - Q_2 \quad (42)$$

$$2\pi\{2[Q_1 + (1 - \epsilon_1)\varphi_{\delta_1}Q_{\delta_1}] - Q_r\} = 4\pi\frac{\sigma_0 T_{\delta_1}^4}{\pi}4\pi(r_1 + \bar{l}_s)^2 \quad (43)$$

$$2\pi\{2[Q_2 + (1 - \epsilon_2)Q_{\delta_2}]\varphi_{\delta_2} + Q_r\} = 4\pi(r_2 - \bar{l}_s)^2 4\pi \frac{\sigma_0 T_{\delta_2}^4}{\pi}. \quad (44)$$

Solving the system of equations (40–44) for Q_r gives:

$$Q_r = \frac{[(Q_1/\epsilon_1)/(r_1 + \bar{l}_s)^2 - \varphi_{\delta 2}(Q_2/\epsilon_2)/(r_2 - \bar{l}_s)^2]}{[1/\epsilon_1 - \frac{1}{2})/(r_1 + \bar{l}_s)^2] + k[1/(r_1 + \bar{l}_s) - 1/(r_2 - \bar{l}_s)] + \{[\varphi_{\delta_2}(1/\epsilon_2 - 1) + \frac{1}{2}]/(r_2 - \bar{l}_s)^2\}}$$

or:

$$Q_r = \frac{[(\sigma_0 T_1^4 4\pi r_1^2)/(r_1 + \bar{l}_s)^2 - (\sigma_0 T_2^4 4\pi r_2^2 \varphi_{2\delta})/(r_2 - \bar{l}_s)^2]}{\{(1/\epsilon_1 - \frac{1}{2})/(r_1 + \bar{l}_s)^2 + k[(1/r_1 + \bar{l}_s) - 1/(r_2 - \bar{l}_s)] + [\varphi_{2\delta}(1/\epsilon_2 - 1) + \frac{1}{2}]/(r_2 - \bar{l}_s)^2\}} \quad (45)$$

Assuming $T_1 = T_2$ and taking into account that in such a case $Q_r = 0$ we have:

$$\frac{r_1^2}{(r_1 + \bar{l}_s)^2} - \frac{r_2^2 \varphi_{2\delta}}{(r_2 - \bar{l}_s)^2} = 0.$$

Hence:

$$\varphi_{2\delta} = \left(\frac{r_1}{r_2} \cdot \frac{r_2 - \bar{l}_s}{r_1 + \bar{l}_s}\right)^2.$$

Consequently:

$$Q_r = \frac{\sigma_0[T_1^4 - T_2^4]4\pi r_1^2}{\{(1/\epsilon_1 - \frac{1}{2}) + (r_1/r_2)^2(1/\epsilon_2 - 1) + k(r_1 + \bar{l}_s)[1 - (r_1 + \bar{l}_s)/(r_2 - \bar{l}_s)] + \frac{1}{2}[(r_1 + \bar{l}_s)/(r_2 - \bar{l}_s)]^2\}}. \quad (46)$$

This formula is valid provided that

$$r_2 > r_1 + 2\bar{l}_s.$$

If $r_2 = r_1 + 2\bar{l}_s$ then equation (46) will be of the type:

$$Q_r = \frac{4\pi r_1^2 \sigma_0 (T_1^4 - T_2^4)}{1/\epsilon_1 + (r_1/r_2)^2(1/\epsilon_2 - 1)}. \quad (47)$$

This formula should be used when

$$r_2 \leqslant r_1 + 2\bar{l}_s.$$

It coincides with the Christiansen formula.

With the help of equation (5) equation (46) can be rewritten as:

$$Q_r = \frac{\sigma_0(T_1^4 - T_2^4)4\pi r_1^2}{(1/\epsilon_1 - \frac{1}{2}) + (r_1/r_2)^2(1/\epsilon_2 - 1) + (kr_1 + 1)[1 - (kr_1 + 1)/(kr_2 - 1)] + \frac{1}{2}[(kr_1 + 1)/(kr_2 - 1)]^2} \quad (48)$$

If $\bar{l}_s$ is small as compared with r_1 and r_2, then equation (46) becomes:

$$Q_r = \frac{\sigma_0(T_1^4 - T_2^4)4\pi r_1^2}{(1/\epsilon_1 - \frac{1}{2}) + (r_1/r_2)^2(1/\epsilon_2 - \frac{1}{2}) + kr_1(1 - r_1/r_2)}. \quad (49)$$

We consider now the simplest problems of composite heat transfer in the immovable grey solid medium, when heat transfer by conduction is commensurable with that by radiation.

We take again two solid planes at constant temperatures T_1 and T_2, where $T_1 > T_2$ (Fig. 6).

Let the emissivities of the planes be ϵ_1 and ϵ_2. Suppose a solid-medium immovable layer of thickness l with a constant thermal conductivity coefficient λ has a constant absorption coefficient k and a coefficient of self-radiation η between the planes. The vector $\mathbf{q}$ of heat transfer is to be

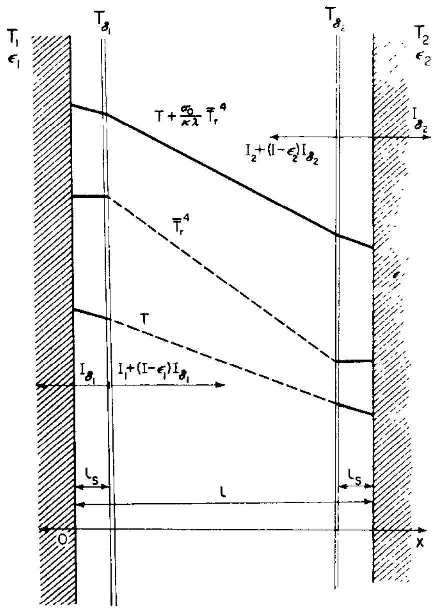

FIG. 6. A calculation scheme of the composite heat transfer in the immovable medium (at the presence of heat transfer by conduction and radiation).

determined. This vector consists of both the thermal conductivity vector $\mathbf{q}_{\text{cond}}$ and the radiation vector $\mathbf{q}_r$, i.e.

$$\mathbf{q} = \mathbf{q}_{\text{cond}} + \mathbf{q}_r.$$

or

$$\mathbf{q} = -\lambda \operatorname{grad}\left(T + \frac{\sigma_0}{k\lambda} T_r^4\right). \quad (50)$$

The following equation holds true for the stationary power state of the medium:

$$\operatorname{div} \mathbf{q} = \operatorname{div} \mathbf{q}_{\text{cond}} + \operatorname{div} \mathbf{q}_r = 0. \quad (51)$$

Substituting expression (50) into this equation gives:

$$\nabla^2\left(T + \frac{\sigma_0}{k\lambda} T_r^4\right) = 0.$$

Assuming the problem considered to be one-dimensional we get:

$$\frac{d^2[T + (\sigma_0/k\lambda) T_r^4]}{dx^2} = 0. \quad (52)$$

Equation (8) with regard for equation (51) becomes:

$$-\operatorname{div} \mathbf{q}_{\text{cond}} = -4k\sigma_0 T_r^4 + 4k\sigma_0 T^4. \quad (53)$$

Equation (2) makes it possible to rewrite this equation as follows:

$$T_r^4 = -\frac{\lambda}{4k\sigma_0} \nabla^2 T + T^4.$$

For the one-dimensional problem we shall have:

$$T_r^4 = -\frac{\lambda}{4k\sigma_0} \cdot \frac{d^2T}{dx^2} + T^4. \quad (54)$$

Substituting equation (54) in equation (52) gives:

$$\frac{d^2[T - (1/4k^2)(d^2T/dx^2) + (\sigma_0/k\lambda)T^4]}{dx^2} = 0$$

Or, after differentiation:

$$-\frac{1}{4k^2}\frac{d^4T}{dx^4} + \left(4\frac{\sigma_0}{k\lambda}T^3 + 1\right)\frac{d^2T}{dx^2} + $$
$$+ 12\frac{\sigma_0}{k\lambda}T^2\left(\frac{dT}{dx}\right)^2 = 0. \quad (55)$$

The solution of this equation could make it possible to find the dependence of the molecule temperature T on the co-ordinate x.

Equation (52) will hold true at any connection between the temperatures T and T_r.

In the particular case when T and T_r are independent, equation (52) breaks into two independent ones:

$$\frac{d^2T}{dx^2} = 0 \quad (56)$$

$$\frac{d^2T_r^4}{dx^2} = 0. \quad (57)$$

The following equalities should be observed in the equlibrium layers close to the wall:

$$T_{\delta_1} = T_{r_1} \;;\; T_{\delta_2} = T_{r_2}. \quad (58)$$

Solving equation (52) and making the solution agree with equation (58) we obtain:

$$T + \frac{\sigma_0}{k\lambda} T_r^4 =$$
$$-\frac{T_{\delta_1} + (\sigma_0/k\lambda)T_{\delta 1}^4 - T_{\delta_2} - (\sigma_0/k\lambda)T_{\delta 2}^4}{kl - 2} \times$$
$$\times (kx - 2) + T_{\delta_1} + \frac{\sigma_0}{k\lambda} T\sigma_{\delta 1}^4. \quad (59)$$

Differentiating this result with respect to x, we shall have:

$$\frac{d[T + (\sigma_0/k\lambda)T_r^4]}{dx} =$$
$$-\frac{T_{\delta_1}(\sigma_0/k\lambda)T_{\delta 1}^4 - T_{\delta_2} - (\sigma_0/k\lambda)T_{\delta 2}^4}{kl - 2} k. \quad (60)$$

For a one-dimensional problem equations (50) and (60) give the following expression:

$$q = \lambda k \frac{T_{\delta_1} - T_{\delta_2}}{kl - 2} + \sigma_0 \frac{T_{\delta 1}^4 - T_{\delta 2}^4}{kl - 2}. \quad (61)$$

Equation (61) is obtained with the aid of equation (52) which, as was mentioned above, will be valid when there is an interdependence between the temperatures T and T_r. Consequently, equation (61) will hold true as well for any interdependence between these temperatures.

Equation (61) shows that the specific heat flux q consists out of the conductive flux q_{cond} and the radiative flux q_r, which are equal to:

$$q_{\text{cond}} = \lambda k \frac{T_{\delta_1} - T_{\delta_2}}{kl - 2} \tag{62}$$

$$q_r = \sigma_0 \frac{T_{\delta 1}^4 - T_{\delta 2}^4}{kl - 2}. \tag{63}$$

It should be noted that the values q_{cond} and q_r are the specific heat fluxes which have gone through a medium layer and they cannot, therefore, be functions of the co-ordinate x.

It follows from equation (61) that the specific heat fluxes q_{cond} and q_r are interconnected since they are both determined by the equilibrium temperatures $T_{\delta 1}$ and $T_{\delta 2}$ other conditions being equal.

Let us formulate the boundary conditions for our problem. The following equations are valid for the equilibrium layers:

$$J_1 - \epsilon_1 J_{\delta_1} = \frac{q_{r1}}{\pi} \tag{64}$$

$$\epsilon_2 J_{\delta_2} - J_2 = \frac{q_{r2}}{\pi}. \tag{65}$$

For these layers the equations of radiant equilibrium will hold true as well:

$$2\pi\left\{2[J_1 + (1 - \epsilon_1)J_{\delta_1}] - \frac{q_{r1}}{\pi}\right\} = 4\pi \frac{\sigma_0 T_{\delta_1}^4}{\pi} \tag{66}$$

$$2\pi\left\{2[J_2 + (1 - \epsilon_2)J_{\delta_2}] + \frac{q_{r2}}{\pi}\right\} = 4\pi \frac{\sigma_0 T_{\delta_2}^4}{\pi}. \tag{67}$$

Solving equations (64) and (66) for $q_r{}^2$ and equations (65) and (67) for q_{r_2} gives the following expressions:

$$q_{r1} = \frac{\sigma_0}{1/\epsilon_1 - \frac{1}{2}} (T_1^4 - T_{\delta_1}^4)$$

$$q_{r_2} = \frac{\sigma_0}{1/\epsilon_2 - \frac{1}{2}} (T_{\delta_2}^4 - T_2^4).$$

As was already mentioned the radiant energy in the layers close to the wall almost does not interact with the medium, in consequence of which we can write:

$$q_{\text{cond}_1} = \lambda k(T_1 - T_{\delta_1})$$

$$q_{\text{cond}_2} = \lambda k(T_{\delta_2} - T_2).$$

K

The boundary conditions for our problem can be finally formulated as

$$q = \lambda k(T_1 - T_{\delta_1}) + \frac{\sigma_0}{(1/\epsilon_1 - \frac{1}{2})} (T_1^4 - T_{\delta_1}^4) \tag{68}$$

$$q = \lambda k(T_{\delta_2} - T_2) + \frac{\sigma_0}{(1/\epsilon_2 - \frac{1}{2})} (T_{\delta_2}^4 - T_2^4). \tag{69}$$

The system of equations (61, 68 and 69) determines the composite heat transfer in a flat layer of the medium. This system is valid provided that $l > 2/k$. It can be solved exactly by the method of successive approximations. It shows that the processes of heat transfer by conduction and radiation are interconnected.

We shall adduce one of the approximate solutions of the system considered.

Using the Lagrange formula equations (61), (68) and (69) can be rewritten:

$$q = \lambda k \frac{T_{\delta_1} - T_{\delta_2}}{kl - 2} + \frac{4\sigma_0}{kl - 2} (T_{\delta_1} - T_{\delta_2})T_x$$

$$q = \lambda k(T_1 - T_{\delta_1}) + \frac{4\sigma_0}{(1/\epsilon_1 - \frac{1}{2})} (T_1 - T_{\delta_1})T_{x1}^3$$

$$q = \lambda k(T_{\delta_2} - T_2) + \frac{4\sigma_0}{(1/\epsilon_2 - \frac{1}{2})} (T_{\delta_2} - T_2)T_{x2}^3$$

where T_x, T_{x_1} and T_{x_2} are some temperatures of the layers close to the wall and of the medium's kernel.

Solving the written system of equations for q gives:

$$q = \lambda \frac{T_1 - T_2}{l} \times$$

$$\times \frac{1}{\begin{array}{l}\{1/[kl + (4\sigma_0 T_{x_1}^3 l)/(1/\epsilon_1 - \frac{1}{2})\lambda]\} + \\ + \{(kl - 2)/[kl + (4\sigma_0 T_x^3 l)/(\lambda)]\} + \\ + \{1/[kl + (4\sigma_0 T_{x_2}^3 l)/(1/\epsilon_2 - \frac{1}{2})\lambda]\}\end{array}}. \tag{70}$$

If the temperature T_1 and T_2 slightly differ from each other, then it can be assumed that:

$$\begin{aligned} T_{x1}^3 &= T_1^3 \\ T_x^3 &= \frac{T_1^3 + T_2^3}{2} \\ T_{x_2}^3 &= T_2^3. \end{aligned} \tag{71}$$

With the help of equation (71), equation (70) enables the value q to be calculated approximately.

Now we shall briefly consider particular cases of our problem.

If $l = 2/k$ then the conductive and radiative fluxes move in the layer independently of each other.

The value q_{cond} is determined by the formula:

$$q_{\text{cond}} = \lambda k \frac{T_1 - T_2}{2}.$$

Since $T_{\delta_1} = T_{\delta_2} = T_\delta$ and $q_{r_1} = q_{r_2} = q_r$ then

$$q_r = \frac{\sigma_0}{(1/\epsilon_1 - \frac{1}{2})}(T_1^4 - T_\delta^4)$$

$$q_r = \frac{\sigma_0}{(1/\epsilon_2 - \frac{1}{2})}(T_\delta^4 - T_2^4).$$

Hence:

$$q_r = \frac{\sigma_0(T_1^4 - T_2^4)}{(1/\epsilon_1) + (1/\epsilon_2) - 1}.$$

Consequently:

$$q = \lambda k \frac{T_1 - T_2}{2} + \frac{\sigma_0(T_1^4 - T_2^4)}{(1/\epsilon_1) + (1/\epsilon_2) - 1}. \quad (72)$$

Equation (72) should be used when $l < 2/k$. If the medium is too solid, then $k \to \infty$, $T_{\delta_1} \to T_1$ and $T_{\delta_2} \to T_2$.

In this case we shall obtain from equation (61):

$$q = \lambda \frac{T_1 - T_2}{l} \quad (73)$$

i.e. heat transfer in very solid media is realized by thermal conductivity alone.

We shall take two cylindrical coaxial surfaces of radii r_1 and r_2. Let the internal and external surfaces have emissivities ϵ_1 and ϵ_2, and the temperatures be constant and equal to T_1 and T_2, where $T_1 > T_2$.

Suppose an immovable grey medium with a constant absorption coefficient k and a radiation coefficient η is enclosed between the cylindrical surfaces. Let us determine the amount of the energy q which goes per unit time through a unit length of the cylindrical medium layer by means of thermal conductivity and radiation.

The stationary power state of the medium for the one-dimensional problem is determined by the equation:

$$\frac{d^2[T + (\sigma_0/k\lambda)T_r^4]}{dr^2} + \frac{1}{r}\frac{d[T + (\sigma_0/k\lambda)T_r^4]}{dr} = 0 \quad (74)$$

The following equalities should be observed in the equilibrium layers:

$$T_{\delta_1} = T_{r_1} \quad ; \quad T_{\delta_2} = T_{r_2}. \quad (75)$$

The solution of equation (74) and its agreement with the equalities (75) gives:

$$T + \frac{\sigma_0}{k\lambda}T_r^4 = \frac{T_{\delta_1} + (\sigma_0/k\lambda)T_{\delta_1}^4 - T_{\delta_2} - (\sigma_0/k\lambda)T_{\delta_2}^4}{\ln[(kr_2 - 1)/(kr_1 + 1)]} \times \ln\frac{kr - 1}{kr_1 + 1} + T_{\delta_1} + \frac{\sigma_0}{k\lambda}T_{\delta_1}^4. \quad (76)$$

The value of q is determined by the expression:

$$q = -\lambda 2\pi r \frac{d[T + (\sigma_0/k\lambda)T_r^4]}{dr}. \quad (77)$$

Having found the product

$$r\frac{d[T + (\sigma_0/k\lambda)T_r^4]}{dr}$$

from equation (76) we substitute it into equation (77) and obtain:

$$q = \frac{T_{\delta_1} - T_{\delta_2}}{(1/2\pi\lambda)\ln[(kr_2 - 1)/(kr_1 + 1)]} + \frac{T_{\delta_1}^4 - T_{\delta_2}^4}{[1/2\pi(\sigma_0/k)]\ln[(kr_2 - 1)/(kr_1 + 1)]}. \quad (78)$$

We can assume that:

$$q_{\text{cond}} = \frac{T_{\delta_1} - T_{\delta_2}}{(1/2\pi\lambda)\ln[(kr_2 - 1)/(kr_1 + 1)]} \quad (79)$$

$$q_r = \frac{T_{\delta_1}^4 - T_{\delta_2}^4}{[1/2\pi(\sigma_0/k)]\ln[(kr_2 - 1)/(kr_1 + 1)]} \quad (80)$$

In the equilibrium layers the relations will be:

$$q_{r_1} = \epsilon_1\sigma_0 T_1^4 2\pi r_1 - \epsilon_1\varphi_{\delta_1}q_{\delta_1} \quad (81)$$

$$q_{r_2} = \epsilon_2 q_{\delta_2} - \epsilon_2\sigma_0 T_2^4 2\pi r_2 \quad (82)$$

$$2[q_1 + (1 - \epsilon_1)\varphi_{\delta_1} q_{\delta_1}] - q_{r_1} = 2\sigma_0 T_{\delta_1}^4 2\pi \frac{kr_1 + 1}{k} \quad (83)$$

$$2[q_2 + (1 + \epsilon_2) q_{\delta_2}]\varphi_{\delta_2} + q_{r_2} = 2\sigma_0 T_{\delta_2}^4 2\pi \frac{kr_2 - 1}{k} \quad (84)$$

where φ_{δ_1} and φ_{δ_2} are the angular coefficients for the corresponding surfaces and q_{δ_1} and q_{δ_2} are the radiations falling upon the corresponding surfaces.

Solving equations (81) and (83) for q_{r_1} and equations (82) and (84) for q_{r_2} we get the expressions:

$$q_{r_1} = \frac{\sigma_0[T_1^4 - T_{\delta_1}^4(kr_1 + 1/kr_1)]}{(1/\epsilon_1 - \frac{1}{2})} \cdot 2\pi r_1 \quad (85)$$

$$q_{r_2} = \frac{\sigma_0[T_{\delta_2}^4(kr_2 - 1)/kr_2 - \varphi_{2\delta} T_2^4]}{\varphi_{2\delta}(1/\epsilon_2 - 1) + \frac{1}{2}} 2\pi r_2. \quad (86)$$

The angular coefficient $\varphi_{2\delta}$ is determined in the following way. Let the conductive heat flux in the medium be absent. Then

$$q_{r_1} = q_{r_2} = q_r.$$

Solving equations (80), (85) and (86) with these approximations for q_r we obtain:

$$q_r = \frac{\sigma_0\{T_1^4[1/(kr_1 + 1)] + T_2^4 \varphi_{2\delta} \times [(r_2/r_1)/(kr_2 - 1)]\}}{\{(1/\epsilon_1 - \frac{1}{2})/(kr_1 + 1) + \ln[(kr_2 - 1)/(kr_1 + 1)] + [\varphi_{2\delta}(1/\epsilon_2 - 1) + \frac{1}{2}]/(kr_2 - 1)\}} 2\pi r_1$$

If $T_1 = T_2$, then $q_r = 0$. From this condition we get:

$$\varphi_{2\delta} = \frac{r_1}{r_2} \cdot \frac{kr_2 - 1}{kr_1 + 1}.$$

Consequently:

$$q_{r2} = \frac{\sigma_0\{T_{\delta 2}^4 \cdot (kr_2 - 1)/kr_2 - T_2(r_1/r_2)[(kr_2 - 1)/(kr_1 + 1)]\}}{(r_1/r_2) \cdot \{[(kr_2 - 1)/(kr_1 + 1)] \times (1/\epsilon_2 - 1)\} + \frac{1}{2}} 2\pi r_2. \quad (87)$$

For layers close to the wall the following expressions will hold true:

$$q_{\text{cond}_1} = \frac{T_1 - T_{\delta_1}}{(1/2\pi\lambda) \ln[(kr_1 + 1)/kr_1)]} \quad (88)$$

$$q_{\text{cond}_2} = \frac{T_{\delta_2} - T_2}{(1/2\pi\lambda) \ln[kr_2/(kr_2 - 1)]}. \quad (89)$$

The boundary conditions for our problem can be finally formulated as:

$$q = \frac{T_1 - T_{\delta_1}}{(1/2\pi\lambda)\ln[(kr_1 + 1)/kr_1]} + \frac{\sigma_0\{T_1^4 - T_{\delta_1}^4[(kr_1 + 1)/kr_1]\}}{(1/\epsilon_1 - \frac{1}{2})} 2\pi r_1 \quad (90)$$

$$q = \frac{T_{\delta_2} - T_2}{(1/2\pi\lambda)\ln[kr_2/(kr_2 - 1)]} + \frac{\sigma_0\{T_{\delta_2}^4[(kr_2 - 1)/kr_2] - T_2(r_1/r_2)[(kr_2 - 1)/(kr_1 + 1)]\}}{(r_1/r_2)[(kr_2 - 1)/(kr_1 + 1)] \times (1/\epsilon_2 - 1) + \frac{1}{2}} 2\pi r_2. \quad (91)$$

The system of equations (78), (90) and (91) determines the composite heat transfer in the cylindrical medium layer. This system is valid provided that $kr_2 - 1 > kr_1 + 1$. It points at an interconnection of the processes of heat transfer by conduction and radiation and is exactly solved by the method of successive approximations.

If $kr_2 - 1 = kr_1 + 1$, then from the system of equations (78), (90) and (91) we get the solution:

$$q = \frac{T_1 - T_2}{(1/2\pi\lambda)\ln(r_2/r_1)} + \frac{\sigma_0(T_1^4 - T_2^4)2\pi r_1}{(1/\epsilon_1) + [(r_1/r_2)(1/\epsilon_2 - 1)]}. \quad (92)$$

The formula just written should also be used when $kr_2 - 1 < kr_1 + 1$.

It should be noted that for very solid media when the products kr_1 and kr_2 become very large, the system of equations (78), (90) and (91) gives the following solution:

$$q = \frac{T_1 - T_2}{(1/2\pi\lambda)\ln(r_2/r_1)} \quad (93)$$

i.e. in very solid media heat is transferred by conduction alone.

Let us now consider heat transfer by thermal conductivity and radiation between the surfaces of concentric spheres of radii r_1 and r_2 under conditions which are analogous to those of heat transfer between the cylindric surfaces.

The amount of heat Q which is transferred per unit time between the sphere surfaces is to be determined.

Solving the one-dimensional equation for stationary heat distribution in the spherical layer gives:

$$Q = \frac{4\pi\lambda(T_{\delta_1} - T_{\delta_2})}{[k/(kr_1 + 1)] - [k/(kr_2 - 1)]} + \frac{4\pi(\sigma_0/k)(T_{\delta_r}^4 - T_{\delta_2}^4)}{[k/(kr_1 + 1)] - [k/(kr_2 - 1)]}. \quad (94)$$

Considerations analogous to those given above make it possible to write the following relations:

$$Q_{\text{cond}} = \frac{4\pi\lambda(T_{\delta_1} - T_{\delta_2})}{[k/(kr_1 + 1)] - [k/(kr_2 - 1)]} \quad (95)$$

$$Q_r = \frac{4\pi(\sigma_0/k)(T_{\delta_1}^4 - T_{\delta_2}^4)}{[k/(kr_1 + 1)] - [k/(kr_2 - 1)]} \quad (96)$$

$$Q_{r_1} = \epsilon_1\sigma_0 \cdot T_1^4 \cdot 4\pi r_1^2 - \epsilon_1\varphi_{\delta_1}Q_{\delta_1} \quad (97)$$

$$Q_{r_2} = \epsilon_2 Q_{\delta_2} - \epsilon_2\sigma_0 T_2^4 4\pi r_2^2 \quad (98)$$

$$2[Q_1 + (1 - \epsilon_1)\varphi_{\delta_1}Q_{\delta_1}] - Q_{r_1} = 2\sigma_0 T_{\delta_1}^4 4\pi \left(\frac{kr_1 + 1}{k}\right)^2 \quad (99)$$

$$2[Q_2 + (1 - \epsilon_2)Q_{\delta_2}]\varphi_{\delta_2} + Q_{r_2} = 2\sigma_0 T_{\delta_2}^4 4\pi \left(\frac{kr_2 - 1}{k}\right)^2. \quad (100)$$

Solving equations (97) and (99) for Q_{r_1}, and (98) and (100) for Q_{r_2}, we obtain:

$$Q_{r_1} = \frac{\sigma_0\{T_1^4 - T_{\delta_1}^4[(kr_1 + 1)/kr_1]^2\}}{(1/\epsilon_1 - \frac{1}{2})} \cdot 4\pi r_1^2 \quad (101)$$

$$Q_{r_2} = \frac{\sigma_0\{T_{\delta_2}^4[(kr_2 - 1)/kr_2]^2 - \varphi_{\delta_2}T_2^4\}}{\varphi_{\delta_2}(1/\epsilon_2 - 1) + \frac{1}{2}} \cdot 4\pi r_2^2 \quad (102)$$

If the conductive heat flux in the medium is absent then from equations (96), (101) and (102) we have:

$$Q_r = \frac{\sigma_0\{T_1^4[1/(kr_1 + 1)^2] - T_2^4\varphi_{2\delta} \cdot [(r_2/r_1)^2/(kr_2 - 1)^2]\}}{(1/\epsilon_1 - \frac{1}{2}) \cdot [1/(kr_1 + 1)^2] + \{[1/(kr_1 + 1)] - [1/(kr_2 - 1)]\} + [\varphi_{2\delta}(1/\epsilon_2 - 1) + \frac{1}{2}][1/(kr_2 - 1)^2]} 4\pi r_1^2$$

Supposing that $T_1 = T_2$ we get:

$$\varphi_{2\delta} = \left(\frac{r_1}{r_2}\right)^2 \cdot \left(\frac{kr_2 - 1}{kr_1 + 1}\right)^2.$$

Consequently:

$$Q_{r2} = \frac{\sigma_0\{T_{\delta_2}^4[(kr_2 - 1)/kr_2]^2 - T_2^4(r_1/r_2)^2[(kr_2 - 1)/(kr_1 + 1)]^2\}}{(r_1/r_2)^2[(kr_2 - 1)/(kr_1 + 1)]^2 \times (1/\epsilon_2 - 1) + \frac{1}{2}} 4\pi r_2^2$$

For layers close to the wall we have the formulae

$$Q_{\text{cond}_1} = \frac{4\pi\lambda(T_1 - T_{\delta_1})}{(1/r_1) - [k/(kr_1 + 1)]} \quad (103)$$

$$Q_{\text{cond}_2} = \frac{4\pi\lambda(T_{\delta 2} - T_2)}{[k/(kr_2 - 1)] - (1/r_2)}. \quad (104)$$

Finally it can be written:

$$Q = \frac{4\pi\lambda(T_1 - T_{\delta_1})}{(1/r_1) - [k/(kr_1 + 1)]} + \frac{\sigma_0\{T_1^4 - T_{\delta_1}^4[(kr_1 + 1)/kr_1]^2\}}{(1/\epsilon_1 - \frac{1}{2})} 4\pi r_1^2 \quad (105)$$

$$Q = \frac{4\pi\lambda(T_{\delta_2} - T_2)}{[k/(kr_2 - 1)] - (1/r_2)} + \frac{\sigma_0\{T_{\delta_2}^4[(kr_2 - 1)/kr_2]^2 - T_2^4(r_1/r_2)^2[(kr_2 - 1)/(kr_1 + 1)]^2\}}{(r_1/r_2)^2[(kr_2 - 1)/(kr_1 + 1)]^2 (1/\epsilon_2 - 1) + + \frac{1}{2}} 4\pi r_2^2. \quad (106)$$

The system of equations (94), (105) and (106) determines the composite heat transfer in a sphere layer thus giving an indication as to the connection between the processes of conduction and radiation.

This system is valid provided that $kr_2 - 1 > kr_1 + 1$ and is exactly solved by the method of successive approximations. If $kr_2 - 1 = kr_1 + 1$, then the following formula holds true:

$$Q = \frac{4\pi\lambda(T_1 - T_2)}{(1/r_1) - (1/r_2)} + \frac{\sigma_0(T_1^4 - T_2^4) \cdot 4\pi r_1^2}{(1/\epsilon_1) + (r_1/r_2)^2(1/\epsilon_2 - 1)} \tag{107}$$

It should be noted that the proper conclusion is again derived, namely that in highly solid media heat is transferred by conduction alone.

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